FISCAL POLICY IN A STOCK-FLOW CONSISTENT (SFC) MODEL: A COMMENT

by

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Abstract
This comment provides a simple analytical exposition of the stock-flow consistent closed economy model used by Godley and Lavoie (2007b) to argue a case for fiscal stabilization policy. We show that the government spending stabilisation rule proposed by Godley and Lavoie (GL) is equivalent to an optimal-output budget deficit rule that automatically ensures budget solvency as long as private sector saving behaviour is itself stable. Assuming a non-inflationary full-employment objective, we derive an optimal government-spending rule.

We endorse GL's view that fiscal policy needs to be "appropriate" if monetary policy is to be actively pursued. The main requirement of fiscal policy is a government debt rule to avoid instabilities arising from the accumulation of debt interest payments. Godley and Lavoie (2007a) simulate such instabilities but do not propose a solution. We do so and derive an optimal monetary rule.

The theoretical substitutability of policy rules raises important questions about the wisdom of macroeconomic stabilization strategies that relegate fiscal policy to a purely supporting role.

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1 Introduction

The once dormant study of fiscal policy as a stabilisation tool has revived in recent years, spurred by the turn of events, notably the formation of Europe's monetary union, and by developments in economic theory. But no agreement has been reached on the virtues of fiscal policy and, in policy circles, monetary policy and central banks remain supreme.

Wynne Godley and Marc Lavoie challenge this state of affairs in a new paper (Godley and Lavoie (2007b)), itself based on their recent book (Godley and Lavoie (2007a)). They argue that fiscal policy could perform a stabilisation role as effectively as monetary policy while ensuring fiscal solvency. The authors further contend that monetary policy is "unable to maintain full employment and low inflation for more than a short period of time" unless fiscal policy is "appropriate". Their key inference is that "the new emphasis on monetary policy may be quite misplaced."

In addition to its criticisms of the consensus view, the authors' work is important from a methodological perspective. Eschewing the mainstream emphasis on micro-founded macroeconomic models populated by optimising households and firms with rational expectations, the authors introduce the notion of a rule-of-thumb target norm between the flow of private income and the stock of private wealth. The authors' approach appears to be in sympathy with Akerlof's critique of mainstream theory, which, he contends, gives insufficient weight to the role that social norms play in economic decision-making (Akerlof (2007)).

Also relevant in this context are the otherwise micro-founded New Keynesian theories that introduce Old Keynesian rule-of thumb consumers into optimising models and thereby find a more significant role for fiscal stabilisation policy.[1] The introduction of rule-of-thumb behaviour comes from a desire by the model constructors to reflect empirical regularities denied by the pure optimising models, notably the importance of *current* disposable income and cash flow as drivers of private expenditure. But to our knowledge, New Keynesian models have not attempted to introduce agents whose rule-of-thumb behaviour is also actively stock-flow consistent.[2]

The purpose of this comment is twofold. First, it provides a simple analytical exposition of the Godley-Lavoie (2007b) closed economy setup.[3] Because of their choice of functional form for the private wealth target, the authors are obliged to argue their case largely by recourse to simulations. These are usefully detailed, but readers might wish for a more summary treatment. A minor

modification of the wealth target specification makes it possible to crystallise the underlying logic.

The second aim is to extend the Godley-Lavoie model to derive an optimal fiscal rule when monetary policy is passive and to define the role that fiscal policy needs to play when monetary policy is optimally active. The results here are not dissimilar to those established in complex optimising models that contain no rule-of-thumb agents.

2 The wealth target and private consumption

Godley and Lavoie (henceforth GL) propose a private sector wealth target of the form:

$$V^* = \varpi Y_d \tag{1}$$

where V is the (end-period) stock of privately held financial wealth and Y_d is private disposable income, both expressed in constant-price terms.[4] Target private wealth, denoted by the superscript asterisk, is a fixed proportion, ϖ, of the private sector's disposable income.

It should be noted that:

- There is no investment in the model so the private sector can be taken to be synonymous with consumers.

- Wealth comprises variable-interest government debt liabilities on which there are no capital gains or losses.[5]

- Consumers are fully non-Ricardian and therefore ignore future taxes required to pay interest on the government debt.

- Equation (1) expresses a "long-run tendency", according to GL, and abstracts from influences, such as capital gains and credit cycles, which will cause temporary fluctuations in the desired financial wealth to income ratio.

Private saving, S, is identically equal to the one-period change in wealth, ΔV, since any excess of disposable income over private consumption must be held in the form of additional holdings of government debt:

$$S \equiv \Delta V \qquad (2)$$

The one-period change in wealth is described by an adaptive process governed by the gap between target wealth and the stock of wealth in the previous period (denoted by the subscript enumerating the lag length):

$$\Delta V = \lambda\left(V^* - V_{-1}\right), \qquad 0 < \lambda < 1 \qquad (3)$$

It follows that the propensity to save out of private disposable income is not a constant, as in the basic Keynesian model, but varies in proportion to the gap between desired and actual financial wealth. In a stationary state, when $V = V^* = V_{-1}^* = V_{-1}$, the propensity to save is zero.

By definition, private consumption, C, is equal to private disposable income less saving, itself equal to the change in the wealth stock:

$$C \equiv Y_d - S \qquad (4)$$

The combination of equations and identities (1) to (4) gives a private consumption function of the form:

$$C = \left(1 - \lambda\varpi\right)Y_d + \lambda V_{-1} \qquad (5)$$

As Godley and Lavoie (2007a) note, the private consumption function thus derived from the wealth target resembles the life-cycle consumption function described by Modigliani (1986). However, within a life-cycle setting, the appearance of *current* rather than *lifetime* disposable income in equation (5) must come from a rule of thumb that equates expected income over remaining lifetimes with current disposable income. Ando and Modigliani (1963) describe this is as their "naïve" hypothesis. Akerlof (2007) explains the relationship between spending and income in terms of a basic social norm that states that current consumption *should* come from current income.

3 Inflation-adjusted private disposable income and budget deficit

In the GL model, private disposable income is adjusted by deducting the "inflation tax" on wealth – broadly equal to the product of inflation and private sector holdings of government debt. The inflation-adjusted budget deficit is correspondingly struck by deducting the same inflation tax. In other respects, the definition of disposable income is conventional.

For the private sector, it comprises the sum of factor income, equivalent to the gross domestic product, Y, and property income, equivalent to the product of the nominal interest rate and the wealth stock $\frac{i}{1+\pi}V_{-1}$[6] - where i is the nominal interest rate and π the rate of inflation - less payments of tax, which are a fixed proportion, ϕ, of factor and property income.

With the inflation tax deducted, private disposable income is approximately equal to the sum of after-tax GDP and interest receipts, the latter calculated using the *real* rate of interest, r. The taxation of nominal interest receipts complicates the arithmetic, however. To simplify, we introduce a measure of the post-tax average real rate of interest, z.[7] Detailed in appendix A, this is defined by:

$$z \equiv \frac{(1-\phi)i-\pi}{1+\pi} \equiv r-\phi\left(r+\frac{\pi}{1+\pi}\right) \tag{6}$$

Inflation-adjusted private disposable income is therefore:

$$Y_d \equiv (1-\phi)Y + zV_{-1} \tag{7}$$

The corresponding inflation-adjusted budget deficit, D, is:

$$D \equiv \Delta V \equiv G + zV_{-1} - \phi Y \tag{8}$$

where G is constant-price government spending, net of debt interest.

4 The role of the interest rate

In stock-flow models it is well known that the rate of interest has an apparently perverse effect on the level of aggregate demand; a higher interest rate *raises* the level of demand as a result of the addition to private interest receipts, equivalent to additional government debt interest payments. This is so even if a higher rate of interest initially raises the rate of saving by sufficient amount to curb private consumption. In stock-flow equilibrium, the debt interest rate effect will always be dominant.[8]

To introduce a negative short-run relationship between private consumption and the rate of interest, GL assume the ratio of target wealth to disposable income depends on the previous-period real rate of interest:[9]

$$\varpi = \omega + \delta r_{-1} \qquad \text{(GL wealth target ratio equivalent)} \qquad (9)$$

This expression treats asymmetrically the roles played by the pre-tax and post-tax real rate of interest. According to equation (9), only the pre-tax rate affects the propensity to save out of disposable income while the post-tax rate affects the level of disposable income, as identity (7) shows. The assumption of a time lag in the impact of the real interest rate also affects the comparison of monetary policy with fiscal policy, to which no time lag is attached. To impose symmetry, we substitute[10]:

$$\varpi = \omega + \delta z \qquad \text{(Amended wealth target ratio)} \qquad (10)$$

The combination of equations (5), (7) and (10) gives a full expression for the consumption function:

$$C = \left(1 - \lambda(\omega + \delta z)\right)(1 - \phi)Y + \left(\lambda + (1 - \lambda(\omega + \delta z))z\right)V_{-1} \qquad (11)$$

The impact on private consumption of a change in the nominal interest rate at unchanged levels of income, wealth and inflation is given by:

$$\frac{dC}{di} = \left(-\lambda\delta(1 - \phi)Y + (1 - \lambda(\omega + 2\delta z))V_{-1}\right)\frac{dz}{di} \qquad (12)$$

Since $\dfrac{dz}{di} > 0$, and the ratio of GDP to the wealth stock varies, derivative (12) will be assuredly negative only if:

$$\omega + \delta z > \frac{1}{\lambda} - \delta z \qquad (13)$$

Following a change in the interest rate, this condition holds if the saving induced by the change in desired wealth geared to interest receipts exceeds the change in interest receipts.

5 Aggregate demand, instruments and stability

Aggregate demand, identically equal to GDP, arises from private consumption and government spending:

$$Y \equiv C + G \qquad (14)$$

The policy-makers' task is to use the available policy instruments to equate aggregate demand with an optimal level of output, denoted by Y^*. In this section, we defer consideration of the optimal level of output and focus on the policy instruments. In the GL model, the tax rate is taken as fixed leaving two instruments: the level of government spending and the nominal interest rate. Granted a suitable degree of inertia in the inflation process, the nominal interest rate can always be manipulated to give a particular post-tax real rate of interest.[11]

The policy-makers' problem can therefore be represented as a choice of G and z that ensures the following equality:

$$Y = f(G, z) = Y^* \qquad (15)$$

Since there are two instruments and one objective, consideration needs to be given to the co-operative role played by each instrument when the other is used actively to equate aggregate demand with the optimal level of output. We examine two options broadly corresponding to those discussed in Godley and Lavoie (2007a and 2007b):

    A. Active fiscal policy, using G; monetary policy passive.
    B. Active monetary policy, using z; fiscal policy passive.

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Formal analysis of the properties of the model explains why the stability of option A depends only on the private sector's behaviour, while option B is intrinsically unstable.[12] With active monetary policy, fiscal policy needs to be adapted to ensure budget solvency – that is, a stable long-run ratio to income of government debt. Solvency is guaranteed under option A, granted stable private sector wealth targeting behaviour.

The stability of the model depends on the dynamic behaviour of the wealth stock, which is itself determined by both private sector saving behaviour and the inter-temporal budget identity.

From the determination of private saving behaviour (equations (1), (2), (3) and (10)) and with output at its optimal level:

$$V = aV_{-1} + bY^*$$
where:
$$a \equiv 1 - \lambda\left(1 - (\omega + \delta z)z\right)$$
$$b \equiv \lambda(\omega + \delta z)(1 - \phi)$$

(16)

The first-order difference equation (16) is stable if $a < 1$, which if true would imply:

$$\omega + \delta z < \frac{1}{z}$$

(17)[13]

The behaviour of the wealth stock at the optimal level of output is also subject to the inter-temporal budget identity. From the definition of the (inflation-adjusted) budget deficit (identity (8)) and the equality of the budget deficit with the change in the wealth stock:

$$V \equiv (1+z)V_{-1} + G - \phi Y^*$$

(18)

Granted a positive z, the first-order difference equation (18) is potentially unstable unless G acts to stabilise the level of debt.[14] The inter-temporal budget identity can therefore be interpreted as a solvency constraint on the behaviour of G.

Under **option A** – active fiscal policy combined with passive monetary policy – G acts in precisely the required fashion, stabilising the level of debt while also ensuring the equality of aggregate demand with the optimal output level. Option

A comprises a nominal interest rate rule that keeps z constant and a stabilisation rule for government spending:

$$G = Y^* - C \tag{19}$$

Under this rule, government spending is set equal to optimal output less private consumption induced by this level of output. From identities (2), (4) and (7)), the identity for private consumption may be written:

$$C \equiv (1 - \phi) Y + z V_{-1} - \Delta V \tag{20}$$

Substitution of identity (20) evaluated at the optimal output level into rule (19) gives:

$$G = \phi Y^* + \Delta V - z V_{-1} \tag{21}$$

The government-spending rule (19) is equivalent to an optimal-output budget deficit rule that guarantees solvency. In particular, government spending is reduced by the build-up of debt interest that would otherwise add to private disposable income and aggregate demand.

Under option A, the wealth stock is entirely determined by private sector saving behaviour and the level of the post-tax real interest rate, as described by equation (16). The latter may be solved forward from an arbitrary set of initial conditions assuming that optimal output grows at a constant rate, κ.

At period n, the wealth stock along this steady growth path is given by:

$$V_n = a^n \left(V_0 - \frac{b(1+k)}{1+k-a} Y_0^* \right) + \frac{b(1+k)}{1+k-a} Y_n^* \tag{22}$$

Assuming stable behaviour with $a < 1$, the first term on the right-hand side goes to zero and in the long-run steady state:

$$\frac{V}{Y^*} \to Lim \frac{V}{Y^*} = \frac{b(1+k)}{1+k-a} \tag{23}[15]$$

Using the definitions in equation (16), the steady-state wealth or debt ratio may be written in full as:

$$Lim \frac{V}{Y^*} = \frac{\lambda(\omega + \delta z)(1-\phi)(1+k)}{k + \lambda(1-(\omega+\delta z)z)} \qquad (24)^{16}$$

Since the debt income ratio stabilises, the debt stock grows at the same rate as optimal output in the steady state. This implies that the primary government surplus (excluding tax raised on debt interest payments) will be equal approximately to the product of the steady-state debt stock and the gap between the post-tax real rate of interest and the growth rate.

Manipulation of the inflation-adjusted budget deficit identity (8) evaluated at the optimal level of output gives:

$$\phi Y^* - G \equiv \left(z - \frac{\Delta V}{V_{-1}} \right) V_{-1} \qquad (25)$$

where the left-hand side is the primary budget surplus.

Since $Lim\left(\dfrac{\Delta V}{V_{-1}}\right) = \kappa$, it follows that:

$$Lim(\phi Y^* - G) = \left(\frac{z - \kappa}{1 + \kappa} \right) Lim(V) \qquad (26)$$

Using equation (24), the steady-state primary budget surplus can be written in full as:

$$Lim(\phi Y^* - G) = (z - \kappa) \left(\frac{\lambda(\omega + \delta z)(1-\phi)}{k + \lambda(1-(\omega+\delta z)z)} \right) Y^* \qquad (27)$$

These solvency results follow directly from the adjustment of government spending according to the optimal-output budget deficit rule (21) and the assumed stability of private sector saving behaviour (17).

Under **option B** - active monetary policy combined with passive fiscal policy – the wealth stock would follow an unstable path were government spending set at an autonomous level, $\bar{G}$. In this case, the inter-temporal budget identity evaluated at the optimal level of output becomes:

$$\Delta V \equiv z V_{-1} + \bar{G} - \phi Y^* \qquad (28)$$

Any small change in the interest rate sets off an explosive rise in debt:

$$d(\Delta V) = dz V_{-1} \tag{29}$$

This result means that passive fiscal policy is incompatible with any attempt by the monetary authorities to implement an active monetary policy.[17] An **option C** is required in which active monetary policy is buttressed by a government debt stabilisation rule.

For the GL model, a debt feedback rule may be written:

$$G = \bar{G} - \mu V_{-1} \quad , \qquad 1 + z > \mu > z \tag{30}$$

Under this rule, the wealth stock would follow a stable path described by:

$$V \equiv (1 + z - \mu) V_{-1} + \bar{G} - \phi Y^* \tag{31}$$

where z and possibly μ are variables.

Studies using optimising models have concluded that the feedback coefficient μ should be only slightly in excess of the real rate of interest, so that the debt stock follows a near random walk. In the model explored by Kirsanova, Stehn and Vines (2005), too large a value for μ creates welfare-reducing cycles in inflation and output, a result of the model's allowance for inflation inertia. In the inflation-inertia-free forward-looking model of Kirsanova and Wren-Lewis (2007), the same conclusion holds for a different reason: short-term changes in government spending incur significant immediate welfare costs that outweigh future costs associated with large changes in the level of government debt.

In the GL model, a different answer applies because the path followed by the wealth stock has to satisfy not only the inter-temporal budget identity but also the private sector's wealth target. The value of μ in equations (30) and (31) must deliver a path for wealth consistent with equation (16).

Detailed in appendix B, the solution is:

$$\mu = \lambda + (1 - \lambda(\omega + \delta z)) z \tag{32}$$

Equation (32) implies that fiscal policy has actively to manage government debt as a counterpart to an active monetary policy. The magnitude of the debt feedback rule, and its excess over the post-tax real rate of interest, varies as the post-tax real rate of interest itself changes in order to align aggregate demand with the optimal output level.

Monetary policy comprises a rule for the nominal rate of interest that gives the following value for the post-tax real rate of interest:[18]

$$z = \frac{1}{\delta\lambda(1-\phi)}\left(\frac{\bar{G}}{Y^*} - \left(\phi + \omega\lambda(1-\phi)\right)\right) \qquad (33)$$

As appendix B shows, the combination of the debt feedback and interest rate rules (equations (30), (32) and (33)) produces an outcome for wealth or government debt under option C (active monetary policy) that is identical to that under Option A (active fiscal policy) evaluated at the same steady-state level of the post-tax real rate of interest.

In sympathy with GL's emphasis on the importance on an "appropriate" fiscal policy to buttress monetary policy, it may be noted that the level of z defined by equation (33) depends on the level of autonomous government spending in relation to optimal output. However, this result is vulnerable to the precise specification of the wealth function and, in particular, to the manner in which substitution effects of interest rate changes are introduced. The robust conclusion is that a rule for debt stabilisation is a necessary counterpart to an active monetary policy and must take account of private sector behaviour.

6 Optimal policy rules

Godley and Lavoie (2007b) add to their closed-economy model a fiscal policy rule related to inflation that intentionally "mimics the various central bank reaction functions that have been proposed since the 1990s." In GL's rule, the growth of government spending is curtailed if the previous rate of inflation is rising or if it exceeds the inflation target. The rule's reaction coefficients are imposed. The authors' aim, which they achieve, is to show that fiscal policy can be as effective as monetary policy in achieving full employment at some target inflation rate.

It is possible to go further and derive the optimal response of government spending and interest rates to inflation developments. Such a derivation helps to

reinforce the authors' view regarding the substitutability of fiscal and monetary policy as stabilisation tools and is of relevance to the debate about fiscal policy rules in member countries of Europe's monetary union. Members do not have access to a monetary policy that can be tailored to their idiosyncratic inflation and output developments.

To devise an optimal rule, it is necessary to make explicit the welfare costs associated with departures of inflation from target or of output from its full-employment level. In micro-based models, the welfare function is derived from the aggregation of consumers' utility. For the current model, it is more appropriate to use a standard quadratic loss function in which undesirable deviations of inflation and output are treated symmetrically and the incremental welfare loss rises with the scale of deviation.

Ignoring discounting, the welfare loss function comprises the weighted sum of the squared deviations of inflation and output, with respective weights β (reflecting the authorities' inflation aversion) and unity:[19]

$$F = \beta\left(\pi - \pi^*\right)^2 + \left(Y - Y^e\right)^2 \tag{34}$$

It is important to note that the activity standard against which output deviations are measured is the equilibrium level of output, Y^e, consistent with the "natural" or "full" employment level of output. Upward inflation bias would arise were political or other pressures to lead to an over-ambitious interpretation of full-employment output. The safeguards that are used to protect monetary policy from such bias – delegation to an apolitical decision-making body with a clear counter-inflation commitment – may not be as easily applied to fiscal policy.
A simple "accelerationist" Phillips curve, similar to that assumed (reluctantly) by GL, describes inflation:

$$\pi = \pi_{-1} + \alpha\left(Y - Y^e\right) \tag{35}$$

The important policy feature of equation (35) is that the government has no direct means to control the rate of inflation. The inflation target has to be pursued indirectly by affecting the level of output.

The optimal trade-off between output and inflation is derived by setting the differential of the welfare loss function with respect to output equal to zero, having substituted for inflation using equation (35):[20]

$$Y - Y^e = -\alpha\beta\left(\pi - \pi^*\right) \qquad (36)$$

It follows that policy makers will seek a level of output equal to the full-employment level only when the inflation gap, $\pi - \pi^*$, is zero. From this trade-off, the optimal level of output can be derived in terms of assumed known variables by substituting for inflation using equation (35):

$$Y^* = Y^e - \frac{\alpha\beta}{1+\alpha^2\beta}\left(\pi_{-1} - \pi^*\right) \qquad (37)$$

Equation (37) shows that the optimal output level is wholly determined by supply characteristics of the economy – its full-employment level and the slope of the short-run Phillips curve, α - by the previous rate of inflation measured against the current target and by the degree of policy-makers' inflation aversion. The greater the degree of aversion, the greater will be the acceptable output sacrifice in response to an inflation shock.

When output rule (37) is applied, inflation follows an optimal path described by[21]:

$$\pi = \pi^* + \frac{\pi_{-1} - \pi^*}{1+\alpha^2\beta} \qquad (38)$$

Under **option A** – active fiscal policy, passive monetary policy – it remains to devise decision rules for the nominal interest rate and for government spending. The monetary rule is straightforward. The monetary authorities use the inflation path shown in equation (38) to set a nominal interest rate that delivers a constant post-tax real interest rate.[22]

For the fiscal authorities, a decision rule can be devised using the path of debt when output evolves optimally. Detailed in appendix C, the rule is:

$$G = aG_{-1} + \left(\phi+b\right)Y^* - \left(\phi a + b\left(1+z\right)\right)Y_{-1}^*$$
where:
$$a \equiv 1 - \lambda\left(1 - \left(\omega + \delta z\right)z\right) \qquad (39)$$
$$b \equiv \lambda\left(\omega + \delta z\right)\left(1 - \phi\right)$$

This rule collapses to the solvency condition (27) in the steady state. Substitution for the current value of optimal output using equation (37) gives the optimal government-spending decision rule as:

$$G = aG_{-1} + (\phi + b)\left(Y^e - \frac{\alpha\beta}{1+\alpha^2\beta}(\pi_{-1} - \pi^*)\right) - (\phi a + b(1+z))Y^*_{-1} \qquad (40)$$

Under **option C** – active monetary policy buttressed by a government debt feedback rule (equation (32)) – the optimal post-tax real rate of interest is similarly derived by substituting for the current value of optimal output in equation (33):

$$z = \frac{1}{\delta\lambda(1-\phi)}\left(\frac{\bar{G}}{Y^e - \frac{\alpha\beta}{1+\alpha^2\beta}(\pi_{-1} - \pi^*)} - (\phi + \omega\lambda(1-\phi))\right) \qquad (41)$$

The monetary authorities use identity (6) and equation (38) to translate this rule into one for the nominal rate of interest.

In contrast to rules with arbitrary reaction coefficients, these optimal rules embed the supply and demand characteristics of the economic model under consideration together with policy-makers' preferences in a way that minimises welfare losses. The rules are naturally only to be regarded as optimal in the context of the GL model and assume co-operative behaviour between the monetary and fiscal policy authorities.

7 Conclusion

This comment has provided a simple analytical exposition of the GL model with a minor modification to the rule-of-thumb private sector wealth target. We have shown that the government spending stabilisation rule proposed in Godley and Lavoie (2007b) is equivalent to an optimal-output budget deficit rule, or what in Old Keynesian models would have been referred to as a full-employment budget rule.

In a stock-flow consistent model, this rule can be shown to have the advantage of automatically ensuring budget solvency as long as private sector saving behaviour is itself stable. We have further shown how an optimal government-spending rule could be devised in the presence of passive monetary policy.

This comment endorses GL's view that fiscal policy needs to be "appropriate" if monetary policy is to be actively pursued. The robust conclusion is that fiscal

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policy must incorporate a government debt rule to avoid instabilities arising from the accumulation of debt interest. Similar conclusions are drawn from optimising micro-based models. Godley and Lavoie (2007a) simulate such instabilities but do not propose a solution. This comment has done so and derives an optimal monetary rule using the GL model.

As in New Keynesian models, the strongest case for fiscal policy as a stabilisation tool emerges in the presence of nominal rigidities and rule-of-thumb consumers that fully populate GL's model. The case for fiscal stabilisation needs in addition to address problems of time-inconsistency and possible inflation bias that may pose difficult institutional challenges. That aside, the theoretical substitutability of policy rules raises important questions about the wisdom of macroeconomic stabilization strategies that relegate fiscal policy to a purely supporting role.

Notes

[1] See, for example, Galí et al. (2007), Muscatelli and Tirelli (2005) and the critique by Ploeg (2005).

[2] Being liquidity-constrained, the rule-of-thumb consumers introduced in New Keynesian models have no wealth. The behaviour of optimising consumers is stock-flow consistent.

[3] The briefly considered open-economy model in Godley and Lavoie (2007b) is not central to their main conclusions.

[4] Godley and Lavoie (2007b) use uppercase (lowercase) letters to denote nominal (constant price) magnitudes. As our exposition is almost entirely in constant price terms, this distinction is not followed here. An appendix summarises our notation and, where different, shows the GL equivalent.

[5] Government debt could be construed as comprising in addition non-interest bearing high-powered money held in fixed proportion within the private wealth portfolio.

[6] The $1+\pi$ term arises from the deflation of the wealth stock by the price level in the previous period.

[7] Robert Rowthorn first suggested this simplification.

[8] Blinder and Solow (1973) were the first to make this point.

[9] GL modify the coefficient on disposable income in the consumption function. Equation (9) is the equivalent expression in terms of the wealth target ratio.

[10] Symmetry could be obtained by the introduction of one-period implementation time lag applied to changes in fiscal policy instruments. The basic point of the analysis would not be affected.

[11] The inflation process that applies under an optimal rule is derived below.

[12] Godley and Lavoie (2007b) do not formally analyse the stability properties but Chapter 11 of Godley and Lavoie (2007a) documents simulated instabilities arising from an active monetary policy.

[13] Using equation (13), the condition for stability and a short-run negative relationship between private consumption and the post-tax real rate of interest is therefore: $\frac{1}{\lambda} - \delta z < \omega + \delta z < \frac{1}{z}$.

[14] We rule out consideration of regimes in which z is negative. Leeper (1991) examines a perverse regime ("passive monetary policy" in his terminology) in which the interest rate is *lowered* in response to an inflationary shock.

[15] This implies that equation (22) could be written:

$$V_n = a^n \left(V_0 - Lim(V_0) \right) + \frac{b(1+k)}{1+k-a} Y_n^*.$$

[16] This corresponds to equation (24) in Godley and Lavoie (2007b) after substituting for z.

[17] Leith and Wren-Lewis (2000) draw the same conclusion (repeated almost verbatim here) using an optimising model.

[18] Were the debt feedback coefficient μ simply set at a constant margin above z, the monetary rule would result in a quadratic expression for the post-tax real interest rate with potentially infeasible solutions.

[19] This textbook model is described in Carlin and Soskice (2005). Note that the welfare function chosen avoids excessive discounting of future outcomes that can cause delayed reaction to inflation shocks.

[20] The substitution gives: $F = \beta \left(\pi_{-1} - \pi^* + \alpha \left(Y - Y^e \right) \right)^2 + \left(Y - Y^e \right)^2$. The derivative with respect to current output is: $\frac{dF}{dY} = 2\alpha\beta \left(\pi - \pi^* \right) + 2 \left(Y - Y^e \right)$. Svensson (1997) shows rigorously how the minimisation of welfare losses over all future periods can be construed as a sequence of one-period welfare loss minimisation problems.

[21] Derived by substituting for the output gap in equation (35) using equation (37).

[22] In a model in which policy instruments act on output with a one-period delay, the monetary authorities would take the current inflation rate as given in the calculation of the required real rate of interest. Although the welfare function would be couched in terms of next-period inflation and output deviations, the resulting policy rules would be similar to those derived here.

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Notation appendix

Our notation is similar to that used by Godley and Lavoie (2007b). Where notation differs, the following list shows the GL usage in square parenthesis:

Y	gross domestic product
G	government consumption or spending (excluding transfers)
C	private consumption [X]
S	private saving
D	budget deficit
V	end-period stock of private wealth (government debt)
Y_d	private disposable income
P	price level
ϕ	tax rate [θ]
r	real rate of interest [rr]
i	nominal rate of interest [r]
π	rate of inflation
z	post-tax real rate of interest
κ	rate of optimal output growth [gr]
Δ	one-period change

Target or optimal values of variables are denoted with an asterisk superscript; for example, π^* denotes the target rate of inflation. Lagged values are denoted by a numerical subscript indicating lag length; for example, V_{-1} denotes the previous end-period wealth stock.

GL's use of parameter numbering is dropped in order to avoid confusion with lag or lead length subscripts. The following substitutions apply (GL symbols on the right-hand side):

$$\varpi \equiv \alpha_3, \qquad \lambda \equiv \alpha_2, \qquad \omega \equiv \frac{1-\alpha_{10}}{\alpha_2}, \quad \delta \equiv \frac{\iota}{\alpha_2}.$$

In addition, δz replaces $\dfrac{\iota r_{-1}}{\alpha_2}$.

GL distinguish nominal values from constant-price values of variables using upper and lower case letters. Our limited use of nominal values relies on a diacritical tilde to denote otherwise constant-price values; for example, "$\tilde{V}$" denotes the nominal wealth stock.

Appendix A: Inflation-adjusted private disposable income

Inflation-adjusted private disposable income comprises after-tax GDP and after-tax interest receipts (the components of disposable income as conventionally defined) less the inflation tax on wealth:

$$Y_d \equiv (1-\phi)Y + \left((1-\phi)i - \pi\right)\frac{\tilde{V}_{-1}}{P} \qquad (A1)$$

The previous-period nominal wealth stock divided by the current price level can be re-expressed thus:

$$\frac{\tilde{V}_{-1}}{P} \equiv \frac{\tilde{V}_{-1}}{P_{-1}}\frac{P_{-1}}{P} \equiv \frac{V_{-1}}{1+\pi} \qquad (A2)$$

The combination of identities (A1) and (A2) gives:

$$Y_d \equiv (1-\phi)Y + \frac{(1-\phi)i - \pi}{1+\pi}V_{-1} \qquad (A3)$$

From the definition of the real interest rate:

$$i \equiv \pi + r(1+\pi) \qquad (A4)$$

Manipulation of identity (A4) gives:

$$\frac{(1-\phi)i - \pi}{1+\pi} \equiv \frac{(1-\phi)\pi + (1-\phi)r(1+\pi) - \pi}{1+\pi} \equiv r - \phi\left(r + \frac{\pi}{1+\pi}\right) \quad (A5)$$

Identity (A5) defines the post-tax real rate of interest:

$$z \equiv r - \phi\left(r + \frac{\pi}{1+\pi}\right) \equiv \frac{(1-\phi)i - \pi}{1+\pi} \qquad (A6)$$

The combination of identities (A3), (A5) and (A6) gives a convenient definition of inflation-adjusted private disposable income:

$$Y_d \equiv (1-\phi)Y + zV_{-1} \qquad (A8)$$

Appendix B: Debt stabilisation and interest rate rules

Under option C, the path of wealth is described by the private wealth adjustment equation and the budget identity with debt feedback:

$$V = aV_{-1} + bY^*$$

where:

$$a \equiv 1 - \lambda\left(1 - (\omega + \delta z)z\right)$$
$$b \equiv \lambda(\omega + \delta z)(1 - \phi)$$

(B1)

$$V \equiv (1 + z - \mu)V_{-1} + \bar{G} - \phi Y^*$$

(B2)

Equations (B1) and (B2) are identical if:

$$\mu = 1 + z - a = \lambda + \left(1 - \lambda(\omega + \delta z)\right)z$$

(B3)

$$(\phi + b)Y^* = \bar{G}$$

(B4)

Equation (B3) is the debt feedback rule while equation (B4) is satisfied as a result of the active use of monetary policy to equate aggregate demand with the optimal level of output. Aggregate demand is defined by:

$$Y \equiv C + G$$

(B5)

$$C = (1 - \phi - b)Y + (1 + z - a)V_{-1}$$

(B6)

$$G = \bar{G} - \mu V_{-1}$$

(B7)

On re-arrangement, the combination of equations (B5) to (B7) gives:

$$Y = \frac{\bar{G} + (1 + z - a - u)V_{-1}}{\phi + b}$$

(B8)

Using the debt feedback rule (B3), equation (B8) becomes:

$$Y = \frac{\bar{G}}{\phi + b}$$

(B9)

Re-arrangement of equation (B9) gives equation (33) for the post-tax real rate of interest required to equate aggregate demand with optimal output. When output is optimal, equation (B9) also satisfies equation (B4).

Appendix C: Optimal government spending rule under option A

From the budget identity, government spending is:

$$G \equiv \phi Y - z V_{-1} + \Delta V \qquad (C1)$$

The path of debt is described by the first-order difference equation:

$$V = a V_{-1} + b Y^*$$
where:
$$a \equiv 1 - \lambda \left(1 - (\omega + \delta z) z \right)$$
$$b \equiv \lambda (\omega + \delta z)(1 - \phi) \qquad (C2)$$

Equation (C2) is used to eliminate terms in the wealth stock from identity (C1) assuming that z is held constant by a passive monetary policy.

The debt difference equation can be conveniently rewritten using the lag operator, L, such that $L^n x \equiv x_{-n}$:

$$V = \frac{b}{1 - aL} Y^* \qquad (C3)$$

Substitution for ΔV and V_{-1} in identity (C1) evaluated at the optimal level of output using equation (C3) gives:

$$G = \phi Y^* - \frac{bz}{1 - aL} Y_{-1}^* + \frac{b}{1 - aL} \Delta Y^* \qquad (C4)$$

Further re-arrangement of equation (C4) provides the rule for government spending:

$$G = a G_{-1} + (\phi + b) Y^* - \left(\phi a + b(1 + z) \right) Y_{-1}^* \qquad (C5)$$